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New paradigm for plasma crystal formation

V N Tsytovich

General Physics Institute, Russian Academy of Science Moscow, Vavilova Street 38, 119991, Moscow, Russia

E-mail: tsytov@lpi.ru

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Abstract

The major properties of complex (dusty) plasmas as a new state of matter are shown to be determined by the collective interaction of two coupled fields, the electrostatic field and the flux field. Both fields determine the grain collective nonlinear screening and grain attraction. Collective interactions together with nonlinear screening are used in the formulation of a new paradigm for plasma crystal formation.

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General description

The collective nature of particle interaction is well established in usual plasmas [1] where interactions between particles and between particles and waves are determined by ‘dressed’ (Yukawa screened) particles with ‘dressing’ of any particle produced by electrostatic field fluctuations of other particles. The pair particle interactions are collective depending on parameters of all other particles. The electrostatic fields \mathbf{E} have both a regular part and a fluctuating part $\mathbf{E} = \langle \mathbf{E} \rangle + \delta \mathbf{E}$. The averaging with respect to the fluctuating field $\delta \mathbf{E}$ describes the collective pair particle collisions, the wave scattering and related induced processes [1]. In the presence of grains the general concept of collective effects survives but is strongly modified. Particularly, the grain interactions can no longer be Yukawa interactions. There are two reasons for that: (1) the grains often have large charges with Z_d about 10^3 – 10^5 and their screening is nonlinear, (2) continuous plasma flux absorption in the process of grain charging is balanced by plasma production only on average and flux fluctuations play an important role. The system containing many grains in its description requires the additional flux field \mathbf{F} . It is remarkable that the concept of plasma flux as a separate field variable was not used previously but it is a rather fruitful concept helping to formulate the physics of open systems using different models for nonlinear screening and for flux fields. The requirement that the nonlinearity in grain screening is small is described by inequality $\beta \equiv \frac{za}{\tau \lambda_{Di}} \ll 1$ where $\tau = T_i/T_e$; $z \equiv Z_d e^2/aT_e$ with $\lambda_{Di} = \sqrt{T_i/4\pi n_0 e^2}$ being the ion Debye length (n_0 is the equilibrium ion density). The condition $\beta \ll 1$ can be fulfilled in astrophysical plasma but

not in most laboratory experiments. For plasma crystal experiments, β ranges from 30 to 80 for radio frequency (RF) discharges [2–4] and direct current (dc) discharges [5], for cryogenic plasmas [6] down to 3–10 in dense plasmas (see citations in the review [7]). The second requirement plays a fundamental role in grain–grain interaction and in the formation of dust self-organized structures. The present consideration only deals with negatively charged grains where the plasma flux is directed to the grain surface and introduces the deposition of material on the grains as observed in most experiments in low-temperature plasmas. Note that in all experiments with positive grains the crystals were not observed so far (see the discussion) and that the theoretical arguments [8] indicate that it is doubtful that they can be created. The plasma flux plays an important role in grain screening and interactions. In general, the flux field has both regular $\langle \mathbf{F} \rangle$ and random $\delta \mathbf{F}$ components: $\mathbf{F} = \langle \mathbf{F} \rangle + \delta \mathbf{F}$. In the absence of external flux the collective grain interaction will be controlled by the fluctuating flux and by fluctuating electrostatic field both for linear and nonlinear screening. Note that the Yukawa interactions cannot be correct even for linear screening. This can be demonstrated by general relations based on linear expansion in perturbations in both fields and based on the assumption that both fields are coupled to each other (the electrostatic field depends on the flux field which in turn depends on the electrostatic field). By introducing electrostatic and flux potentials $\mathbf{E} = -\partial\phi/\partial\mathbf{r}$, $\mathbf{F} = -\partial G/\partial\mathbf{r}$ and extracting the usual Coulomb factor $-Z_d e/r$ from both potentials $\phi = (-Z_d e/r)\psi$, $G = -(Z_d e/r)g$, for spherical symmetry one finds (dependence on the distance r from the grain)

$$\frac{d^2\psi}{dr^2} = c_1\psi + c_2g; \quad \frac{d^2g}{dr^2} = c_3g + c_4\psi \quad (1)$$

where $c_{1,2,3,4}$ are some constants (see the examples in [9–11]). The most important fact is the existence of binding of two fields by these constants. Equations (1) follow from the Poisson equation for the electrostatic field and from the continuity equation for the flux field. The right-hand sides of (1) depend on the source of ionization compensating in equilibrium the absorption on grains. For linear disturbances the right-hand side of (1) is always linear in ψ and g . The coupled equations (1) will give, for the screening factor of the potential, an equation of fourth order in derivatives with respect to distance and therefore the screening of the electrostatic potential and the flux potential in the case where the nonlinearities are neglected *should be described by two exponents but not by one*. Neglecting the contribution of the flux potential in the first equation (1) we have the usual Debye screening $c_1 \approx 1/\lambda_{Di}^2$, while neglecting the effect of the electrostatic field in the second equation (1) we find that c_3 is related to the mean free path for flux absorption on grains. The coupling of two fields changes the screening of both the electrostatic potential and the flux potential. The two terms can have exponent factors containing imaginary contributions [9, 10] and the coefficient in front of the exponents cannot be always positive [11]. This corresponds to over-screening or grain collective attraction. The latter was indeed found in [9, 10] for ionization sources proportional to the electron density and in [11] for the ionization source independent of the electron density. The coupling of the two fields is determined by the Havnes parameter $P = Z_d n_d / n_0$ (with n_d being the dust density) and vanishes in the limit $P \rightarrow 0$. P ranges as $0 < P < 1$ and is about 0.9–0.5 in most experiments. The Havnes parameter also determines other important parameters: (1) the self-energy density $n_0 T_i P z / \tau$ of dust grains supported by the flux (being specially large for $z/\tau \gg 1$ as found in most laboratory experiments), (2) the mean free path for flux absorption being about λ_{Di}^2 / aP . Equation (1) for the flux is valid for the size of the systems much larger than the mean free path for flux absorption on grains (as in most laboratory experiments and in astrophysical conditions). The flux fluctuation contributes to dust–dust attraction and to dust stochastic heating while the ram pressure of external averaged

flux $\langle \mathbf{F} \rangle$ contributes to dust cloud self-contraction and to the formation of dust self-organized structures such as dust voids and dust vortices.

Formulation of a new paradigm for plasma crystal formations

Formulation of a new paradigm uses the nonlinearity in screening and in the effects of electrostatic field and flux field coupling. Together these effects give new qualitative features. The collective attraction was previously investigated for the linear limit [8–11]. The standard concept of a probe particle embedded in dusty plasmas is used for determining the collective pair grain interactions. The result of considering together both the nonlinear screening and the collective interactions caused by flux perturbations shows that the dust attraction of negatively charged grains is a consequence of their self-consistent treatment as disturbances of the charge and flux balance by the probe grains. This is an essential step in understanding the physics of nonlinear collective attraction as being universal and gives some excuse in the formulation of this point as a new paradigm. The results are obtained by numerical calculations of probe grain screening and can be summarized in figure 1(a), where ψ_m is the value of the screening factor at the bottom of the attraction well and r_m is the position of the attraction well. The figure summarizes important general features found in all sets of numerical solutions for nonlinear screening together with collective attraction for different ionization sources. Those are: (1) the collective attraction start to operate at distances where the nonlinearity in screening is small, (2) at distances larger than this distance the collective effects are found to be small, allowing us to use their linear treatment even for strong nonlinearity, (3) the minimum of the potential corresponds to weak attraction potential well $|\psi_m| \ll 1$, (4) the position of the attraction minimum r_{\min} corresponds to distances larger than λ_{Di} (about 6–10 λ_{Di}). The actual curves for the potential depend on the degree of the nonlinearity in screening ν (see below), on parameter β and on the type of ionization source; but for strong nonlinearity and $\beta \gg 1$, an attraction well was always found with $\psi_m \neq 0$. The aim of the present paradigm is to demonstrate that the values which can be easily estimated from the measured data: the critical value of the coupling constant Γ_{cr} for non-screened grains, the inter-grain distance r_{\min} and the grain phase transition temperature $T_{d,cr}$, can be explained both for RF and dc discharges and for cryogenic plasmas by a reasonable choice of a single parameter, the mean free path for ion–neutral collisions. In the present concept, the strong grain repulsion exists only at short distances and the grains spent most of the time at large distances, close to the minimum of the potential well where the interaction is weak. The interactions of grains became similar to molecular interactions and the coupling constant can be found from the Lindeman criterion [12]

$$\Gamma_{cr} = \frac{Z_d^2 e^2}{r_{\min} T_{d,cr}} = \frac{1}{|\psi_m|}. \quad (2)$$

Relation (2) physically means simply that the temperature of grains is equal to the deepness of the collective attraction well. Γ_{cr} defined by the first expression (2) is a *ratio of the 'fictive' non-screened Coulomb energy to the thermal energy of grains*. As can be seen from figure 1(a), the grain field is not only completely screened at certain distance but is over-screened so that the polarization charge for large distances has that opposite sign to that for small distances which causes the grain attraction. All calculations performed so far indicate that $|\psi_m| \ll 1$ and that the new paradigm is able to explain the large values of Γ_{cr} (about 10^3 – 10^4 observed in existing RF and dc and in cryogenic experiments [3–7]) which was not possible to explain in any approach using the concept of strong grain interactions. According to [9, 10, 13] the equilibrium state in dusty plasmas should be determined by two conditions: not only by the single condition of charge quasi-neutrality, but also by another condition of balance of flux

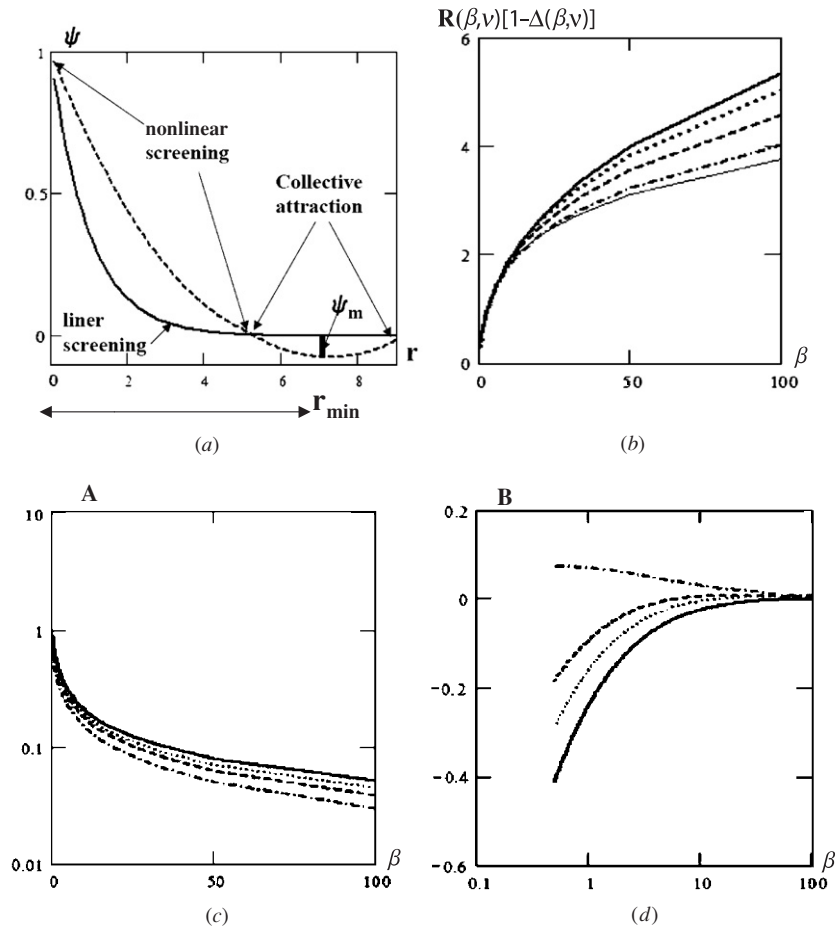


Figure 1. (a) Schematic drawing of dependence of the screening factor ψ on the distance r in units of the ion Debye length λ_{Di} ; (b) radius of nonlinear screening R_{nl} as a function of β for different values of ν , the solid thick line corresponds to $\nu = 0.1$, the dotted line corresponds to $\nu = 0.3$, the dashed line corresponds to $\nu = 0.5$, the dash-dotted line corresponds to $\nu = 0.7$ and the thin solid line corresponds to $\nu = 0.9$; (c) and (d) the coefficients A and B respectively entering the collective screening factor as a function of β , the solid thick line corresponds to $\nu = 0.1$, the dotted line corresponds to $\nu = 0.3$, the dashed line corresponds to $\nu = 0.5$ and the dash-dotted line corresponds to $\nu = 0.9$.

absorption and flux creation through ionization. One can therefore expect that the perturbations of this balance by a probe grain will depend on the ionization source. Two models were used in linear approach, one for an ionization source proportional to the electron density [9, 10] (most often found in experiments) and another for an ionization source independent of the electron density [11]. In both cases, the physical reason for the appearance of attraction is that the source creates electron-ion pairs between two negatively charged grains with electrons being able to leave this region rather fast, and the ions not, having friction both on grains and on neutral atoms. For strong nonlinear screening, $\beta \gg 1$, the friction of ions on grains rapidly decreased with increasing β (as shown recently in [14]). Therefore, here we demonstrate the new paradigm for the most interesting case where the ion-neutral collisions dominate ion friction.

Nonlinear screening and modification of collective interactions

For $\tau \ll 1$ and for an ionization source proportional to the electron density, the collective part of the screening factor $\psi_{\text{coll},e}$ contains a completely imaginary factor in the argument of the second exponent while for a source independent of the electron density, the factors in the argument of the exponent of $\psi_{\text{coll},c}$ are both real: $\psi_{\text{coll},e} = A_e \exp(-\lambda_1(r - R(s)) + B_e \cos(\lambda_{2,e}r))$; $\lambda_1 = \sqrt{k_0^2 + 1 + \frac{1}{1+P}}$; $\lambda_{2,e} = \frac{k_0}{\lambda_1} \sqrt{\tau P}$; $\psi_{\text{coll},c} = A_c \exp(-\lambda_1(r - R(s)) + B_c \exp(-\lambda_{2,c}(r - R(s)))$; $\lambda_{2,c} = \frac{k_0}{\lambda_1} \sqrt{\tau(1 - P + \frac{(1+P)}{z})}$ where $k_0^2 = zPa/2\sqrt{\pi}(1+z)\lambda_{\text{in}}$ with λ_{in} being the mean free path for ion–neutral collisions [9, 11]. The difference between $\lambda_{2,e}$ and $\lambda_{2,c}$ is not very important, both of them contain a small factor $\sqrt{\tau}$ and the expressions under the square root are not very different. The second terms are completely collective and vanish in the limit $P \rightarrow 0$. We use the results, supported by the investigation summarized in figure 1(a), that the collective effects can be considered to be linear, and therefore expressions under the exponents are the same for the linear ($\beta \ll 1$) limit and the nonlinear ($\beta \gg 1$) limit but the amplitudes A and B depend on boundary conditions and they are different in the linear and nonlinear limits. For $\beta \ll 1$, the boundary conditions are determined by the field values at the grain surface ($R(s) = a$) while for $\beta \gg 1$ they are determined by the conditions for joining the potential and fields with the nonlinear screening $R(s) = R_{\text{nl}}$ (the position where ψ in figure 1(a) is close to zero); R_{nl} was calculated numerically using explicitly the properties of nonlinear screening described in detail in [14] for arbitrary ion polarization charge $\rho_i \propto \phi^\nu$; $0 < \nu < 1$; $\tau \ll 1$ by exactly solving the Poisson equation and fitting the results with simple analytical expressions $\psi_{\text{nl}} = (1 - \frac{r}{R(\beta, \nu)})^{2/(1-\nu)}$ where the nonlinear screening radius $R(\beta, \nu)$ given by numerical fitting with a good accuracy as a function of β and ν is $R(\beta, \nu) = d(\nu)\beta^{(1-\nu)/(3-\nu)}$; $d(\nu) \approx 1.9 + 12\nu^3$ (the power independence of R from β is exact and the factor in front of it is found here by fitting numerical curves obtained in [14]). The nonlinear screening radius for joining the nonlinear and collective screening potentials $R_{\text{nl}}(\beta, \nu) = R(\beta, \nu)(1 - \Delta(\beta, \nu))$ is found by the numerical solution of a transcendental equation $\Delta(\beta, \nu)^{2/(1-\nu)} = (1 - \Delta(\beta, \nu))d(\nu)\beta^{-2/(3-\nu)}$ and the numerical results are presented in figure 1(b). Joining the nonlinear ψ with collective ψ at $r = R_{\text{nl}}$ gives the full screening curve (see figure 1(a)) for different β and ν . The coefficients A and B behave similarly for the first and second types of ionization sources since $\lambda_{2,c}R_{\text{nl}}$ is $\ll 1$. Figures 1(c) and (d) show the results of numerical computations of coefficients A and B . B is negative and gives an attraction well close to R_{nl} .

Position of the first minimum of potential energy

After obtaining the total screening curve, it is possible to calculate the position of the minimum of the first attraction well for both types of ionization sources. With an increase of β the absolute value of negative B decreases and this potential well disappears. For a source proportional to the electron density the first minimum should then occur only for much larger distances where the cos-term changes its sign. The results of numerical computations for this case are shown in figure 2(a). For a source independent of the electron density (with second exponent instead of cos-term), the attraction well for large β disappears (the numerical results are shown in figure 2(b)). It is easy to see that for β about 30–80, for the experiments on plasma crystals in RF and dc discharges, this position is about $(7-10)\lambda_{\text{Di}}$; since λ_{Di} is typically $35 \mu\text{m}$, the inter-grain separation is about 200–250 μm in rough agreement with the values observed.

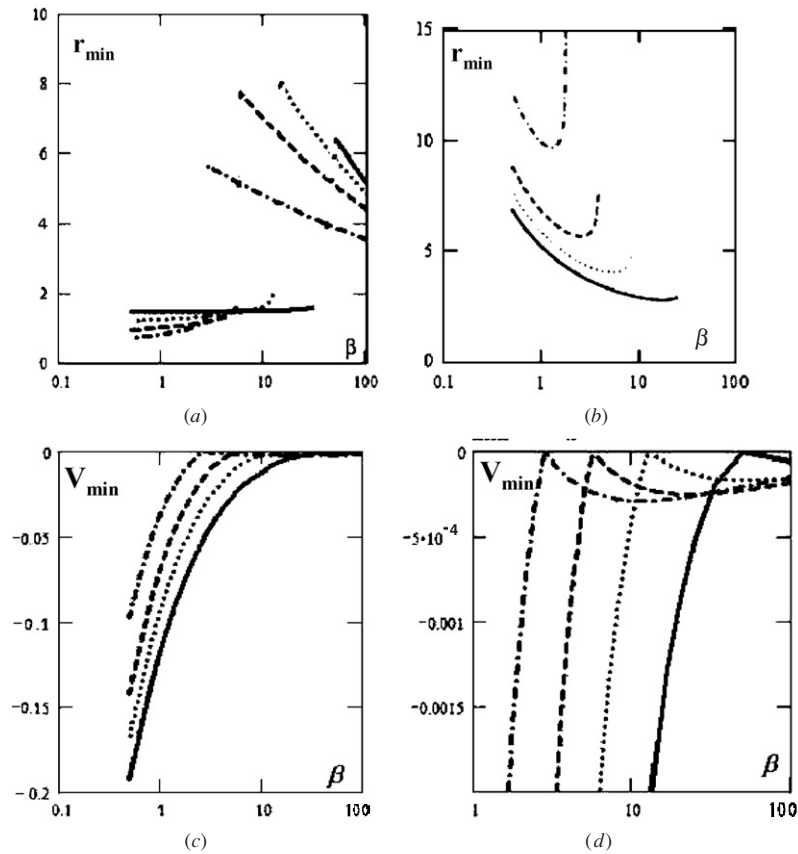


Figure 2. (a) Position of the minimum of the attraction well in units of R_{nl} for the ionization source proportional to the electron density; (b) position of the minimum of the attraction well in units of R_{nl} for the ionization source independent of the electron density; (c) absolute value of the energy of the minimum of the attraction well in units of $Z_d^2 e^2 / R_{nl}(\beta, \nu)$ for $\beta < 10$; (d) same as in (c) but with smaller vertical axis scales and in the range $\beta < 100$; for all figures the solid line corresponds to $\nu = 0.1$, the dotted line corresponds to $\nu = 0.3$, the dashed line corresponds to $\nu = 0.5$ and the dash-dotted line corresponds to $\nu = 0.7$.

The binding energy of the first attraction well

Using the full-screened potential we find the absolute value of the negative energy of the first potential well for both types of ionization sources. The value of this potential well determines the dust temperature for the phase transition (figures 2(c) and (d)). The typical binding energy is about 5–15 eV close to that observed in experiments after crystal melting. In the present consideration, the gradients of external potential in the range of distances of attraction well are considered to be small—the additional ion drifts produced by them should not interfere with the drift in grain interaction and screening. We exclude from consideration the case of large ion drift produced by large external electric fields.

The coupling constant

The value of the coupling constant Γ_{cr} was calculated numerically using expression (2). The examples of results are given in figures 3(a) and (b) for the case of the ionization source

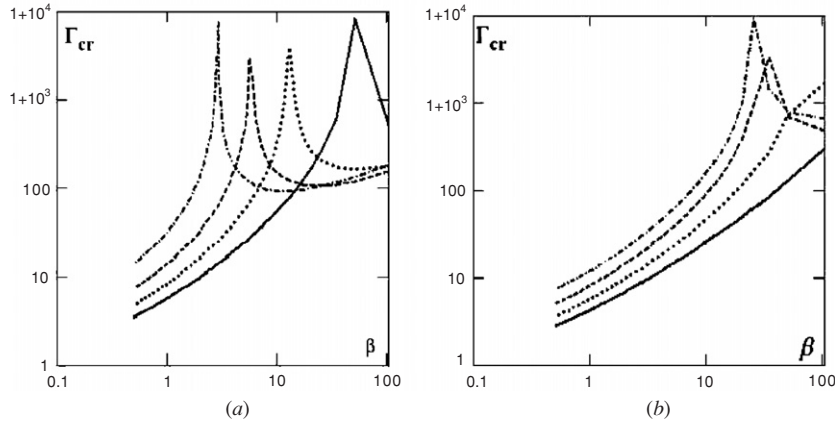


Figure 3. (a) Γ_{cr} as a function of β and ν for $\tau = 0.1$, $P = 0.9$, the solid line corresponds to $\nu = 0.1$, the dotted line corresponds to $\nu = 0.3$, the dashed line corresponds to $\nu = 0.5$ and the dash-dotted line corresponds to $\nu = 0.7$; (b) the same as in (a) but for $\tau = 0.03$ and $P = 0.26$.

proportional to the electron density. It should be mentioned that this figure illustrated for the first time that the predicted values of Γ_{cr} can be, for β about 30–80, as high as 10^3 or 10^4 , of the order of that observed in RF and dc discharges [3–5] and for cryogenic discharges [6, 7]. It also predicts that the crystals can exist for β about 3–10 with Γ_{cr} about 4–10 (see below for the discussion of the possibility of crystal formation in dense or thermionic plasmas).

Flux at the surface of plasma crystals

It was not noted earlier that the properties of crystal surface layer with the thickness of the order of the flux mean free path are rather complicated. It is indeed the case since any finite-size grain collection creates a regular flux on its surface and in the surface layer both the regular flux and the random flux are present. The calculation of surface effects should be addressed in future by taking into account that the grain interaction is continuously changing from collective attraction to non-collective shadow attraction (about the latter see [15, 16]). Numerical calculations and some experiment indicate that the drift ion velocity u_i of the regular flux on the surface of grain collection is larger than the ion thermal velocity ($u = u_i / \sqrt{2T_i/m_i}$ about 2–5). An estimate of additional force F_s acting on grains at the surface S of the crystal can be found by dividing the total flux on the surface, $2u^2 S n_i T_i$, by the number of grains in the surface layer of the thickness of the mean free path $n_d S \lambda_{\text{Di}}^2 / a P$. We find $F_s \approx 8\pi z T_e u^2 n_i a^2$ which is estimated to be comparable or larger than the existing external confinement force. These forces can be responsible for the surface tension.

Flux fields in gaseous dusty plasmas

In a gaseous state, the fluctuations of grain charge create a new phenomenon—non-conservation of grain kinetic energy in grain–grain collisions. In each collision only the sum of kinetic energy and self-energy (depending on grain charges) is conserved. This effect is responsible for stochastic grain heating with an increase of the average grain kinetic energy E_d [17] estimated as $dE_d/dt \approx 4\pi E_d n_d a^3 \omega_{\text{pd}}^2 / \omega_{\text{pi}}$ where ω_{pd} and ω_{pi} are the dust plasma and ion plasma frequencies, respectively. Excitation of the regular flux creates universal

instabilities [13] which results in the formation of self-organized structures such as dust voids [18, 19] and dust vortices.

Discussions

We first discuss the validity of the procedure used to join the nonlinear and collective screening factors. In [12], it was proposed to solve directly together the nonlinear equations both for the screening and the flux. Such equations were formulated for certain assumptions and examples of their solution were given in [12]. This procedure was shown to be rather complicated and non-productive, but it was noted that the case of the interference of nonlinear and collective effects is exceptional. The main progress obtained at present is that a small parameter k_0^2 was found responsible for the collective effects to operate at large distances and to be weak at small distances. It is used for the separation of two regions, the distances where the nonlinear effects are operating and the distances where the collective effects start to operate with correspondent matching of two solutions. It was checked that the exact value of the matching point does not much affect the results and that the most important of the collective effects is that at the joining distance the nonlinearity provides large gradients of the screening factors. The new effects found to be introduced by the nonlinearity finally allow us to obtain the values of three parameters Γ_{cr} , r_m , $T_{d,cr}$ in reasonable agreement with observations (the three parameters are determined only by one, a reasonable value of which can also be chosen from observations). This procedure shows its effectiveness. Future investigations can be made without using the joining procedure but it is clear that the collective effects start only at large distances where the nonlinearity becomes small and therefore the separation used has definite physical meaning. Crystal formation was not observed so far for positive grains, emitting electrons either thermionic or due to external ultraviolet radiation [20], although the calculated Γ was large [21]. It is noted in [21] that both negative and positive charges can be found in dense plasmas. The present calculations show that crystals can be formed for negative grains and for relative low value Γ_{cr} of about 7–30 is found in [21] and efforts could be made to observe such crystals. For positive grains, as shown in [8], the force balance for dust clouds cannot be fulfilled and the self-confinement discussed above is absent. For thermionic positive grains, the grain attraction was recently found in [22, 23]. Future investigations should include the heat flux effects in grain interaction not considered in [22, 23]. The heat flux to the grains is necessary to support their high temperature for thermionic emission. This can create strong thermophoretic forces in the attraction of grains (according to [24]), the collective effect in heat flux shadowing and nonlinear screening. More detailed analysis of the description of collective effects and screening in dusty plasmas with positive grains could be made using the present concept of two interacting fields. The present paper does not pretend to describe the case of positively charged grains but indicates the scenario for future theoretical investigations.

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